

# Coupled Borrows

## Expanding Core Proof Inference in Prusti

July 2023

# Rust, Prusti, and Memory Safety

Rust compiler guarantees **memory safety**.

- Affine Types
- “Aliasing XOR Mutability”
- Ownership, Borrowing

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## Prusti

Decidable memory safety analysis  $\rightsquigarrow$  automated **core proof**

# Rust, Prusti, and Memory Safety

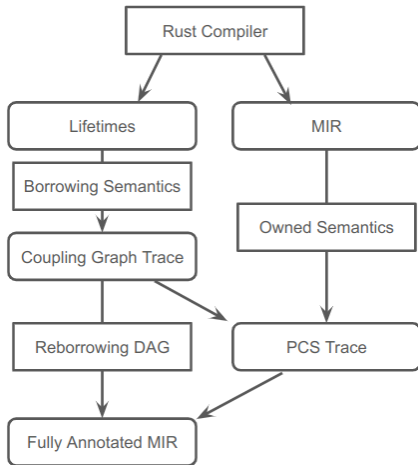
Released version of Prusti has limited support for

- Reborrowing inside loops
- Shared borrows
- Reborrowing at function boundaries
- Borrows inside structs

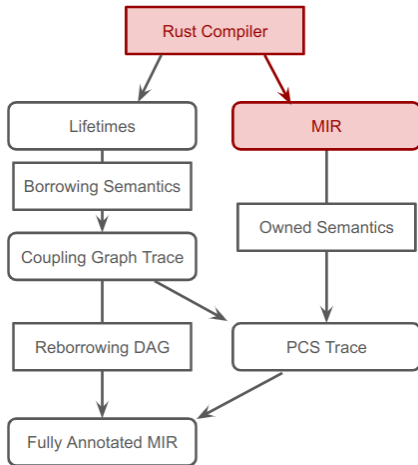
## Main Problem

How can we extend Prusti's *model of Rust* to automatically support these features going forward?

# Overview



# Overview



# Background: MIR

**MIR:** Mid-level Intermediate Representation

**Place:** Represent accessible memory as (local, [projection]).

- **locals:** arguments, locally declared variables: `_1`, `_2`, ...
- **projection:**
  - dereference (`*_1`)
  - field access (`_2.0`)
  - enum variant downcast (`_3 as Some`)
  - more including indexing, slices, type casts ...



# 1<sup>st</sup> notion of memory safety



## Memory Safety in Rust

Fixed point analyses over MIR.

Ensures **definite accessibility** before use.

⇒ No use after free errors.

Result of definite accessibility analysis.  
Traces accessibility through the MIR.

# 1<sup>st</sup> notion of memory safety



## Memory Safety in Rust

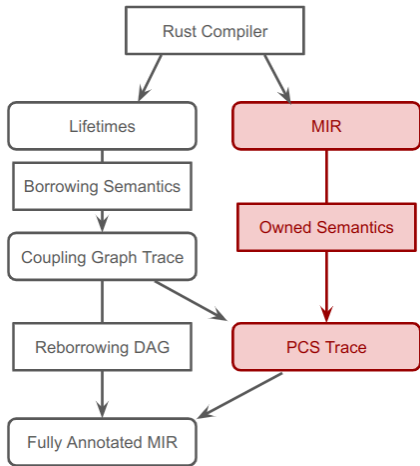
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Ensures definite accessibility before use.

⇒ No **use after free** errors.

Result of definite accessibility analysis.  
Traces accessibility through the MIR.

# Overview



## Setting: Ownership

```
struct T {}  
fn main() {  
    let t1: T;  
    let mut t2 = T {};  
    t1 = t2;  
}
```

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```

### Prusti's Approach to Explaining Owned Data

- 1 Model program state as *capabilities* to memory.
- 2 Describe *rules* in terms of that state.
- 3 Exploit *approximations* from the type system.

# Capabilities

Things we can *do* with places: **Capabilities**.

Program state (all places): **Place Capability Summary (PCS)**.

Capability		Read?	Write?	Nonaliasing?	Example
<i>exclusive</i>	E p	yes	yes	yes	let mut p = ();
<i>immutable</i>	R p	yes	no	yes	let p = ();
<i>shared</i>	S p	yes	no	no	*p where p:&()
<i>write</i>	e p	no	yes	yes	let mut p:();
<i>allocated</i>	r p	no	no	yes	drop(p); p

# Capabilities

Things we can *do* with places: **Capabilities**.

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<i>write</i>	e p	no	yes	yes	let mut p:();
<i>allocated</i>	r p	no	no	yes	drop(p); p

Missing rows:

- No permissions.
- Write to aliased memory.

## Example: Owned Data

```
StorageLive(1);  
StorageLive(2);
```

```
2 = T{};
```

```
1 = move 2;
```

```
0 = ();
```

```
StorageDead(2);
```

```
StorageDead(1);
```

```
return;
```

```
struct T {}  
fn main() {  
    let t1: T;  
    let mut t2 = T {};  
    t1 = t2;  
}
```



## Example: Owned Data

```
//           { e 0 }  
StorageLive(1);  
StorageLive(2);  
  
2 = T{};  
  
1 = move 2;  
  
0 = ();  
  
StorageDead(2);  
  
  
StorageDead(1);  
  
return;
```

```
struct T {}  
fn main() {  
    let t1: T;  
    let mut t2 = T {};  
    t1 = t2;  
}
```

Rule:  
allocate return place as e

## Example: Owned Data

```
//           { e 0 }  
StorageLive(1);  
StorageLive(2);  
//           { e 0, e 1, e 2 }  
2 = T{};  
  
1 = move 2;  
  
0 = ();  
  
StorageDead(2);  
  
  
StorageDead(1);  
  
return;
```

```
struct T {}  
fn main() {  
    let t1: T;  
    let mut t2 = T {};  
    t1 = t2;  
}
```

Rule:

```
{ } StorageLive(1) { e 1 }  
{ } StorageLive(2) { e 2 }
```

## Example: Owned Data

```
//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
1 = move 2;

0 = ();

StorageDead(2);

StorageDead(1);

return;
```

```
struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}
```

Rule:

```
{ e 2 }
  2 = (const)
{ E 2 }
```

## Example: Owned Data

```

//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
1 = move 2;
//           { e 0, R 1, e 2 }
0 = ();

StorageDead(2);

StorageDead(1);

return;

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{E 2, e 1}
  1 = move 2
{e 2, R 1}

```

## Example: Owned Data

```

//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
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//           { e 0, R 1, e 2 }
0 = ();
//           { E 0, R 1, e 2 }
StorageDead(2);

StorageDead(1);

return;

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{e 0}
  0 = (const)
{E 0}

```

## Example: Owned Data

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StorageLive(1);
StorageLive(2);
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//           { e 0, e 1, E 2 }
1 = move 2;
//           { e 0, R 1, e 2 }
0 = ();
//           { E 0, R 1, e 2 }
StorageDead(2);
//           { E 0, R 1 }

StorageDead(1);

return;

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{ e 2 }
  StorageDead(2)
{ }

```

## Example: Owned Data

```

//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
1 = move 2;
//           { e 0, R 1, e 2 }
0 = ();
//           { E 0, R 1, e 2 }
StorageDead(2);
//           { E 0, R 1 }
//           drop(1)
//           { E 0, r 1 }
StorageDead(1);

return;

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{ R 1 }
    drop(1)
{ r 1 }

```

## Example: Owned Data

```

//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
1 = move 2;
//           { e 0, R 1, e 2 }
0 = ();
//           { E 0, R 1, e 2 }
StorageDead(2);
//           { E 0, R 1 }
//           drop(1)
//           { E 0, r 1 }
StorageDead(1);
//           { E 0 }
return;

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{r 1}
  StorageDead(1)
{}

```



## Example: Owned Data

```

//           { e 0 }
StorageLive(1);
StorageLive(2);
//           { e 0, e 1, e 2 }
2 = T{};
//           { e 0, e 1, E 2 }
1 = move 2;
//           { e 0, R 1, e 2 }
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//           { E 0, R 1, e 2 }
StorageDead(2);
//           { E 0, R 1 }
//           drop(1)
//           { E 0, r 1 }
StorageDead(1);
//           { E 0 }
return;
//           { }

```

```

struct T {}
fn main() {
    let t1: T;
    let mut t2 = T {};
    t1 = t2;
}

```

Rule:

```

{E 0}
  return
{ }

```

## Example: Packing and Unpacking

```
struct T {}  
struct S { f: T, g: T }  
  
StorageLive(s);  
StorageLive(x);  
s = S {f: T{}, g: T{}};  
//                               { E s, e x }  
  
x = move s.g;
```

## Example: Packing and Unpacking

```
struct T {}  
struct S { f: T, g: T }  
  
StorageLive(s);  
StorageLive(x);  
s = S {f: T{}, g: T{}};  
//           { E s, e x }  
//           unpack(E s)  
//           { E s.f, E s.g, e x }  
x = move s.g;
```

Rule:

```
{ E s }  
  unpack(E s)  
{ E s.f, E s.g }
```

## Example: Packing and Unpacking

```
struct T {}
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StorageLive(s);
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s = S {f: T{}, g: T{}};
//           { E s, e x }
//           unpack(E s)
//           { E s.f, E s.g, e x }
x = move s.g;
//           { E s.f, e s.g, R x }
```

## Example: Packing and Unpacking

```
struct T {}
struct S { f: T, g: T }

StorageLive(s);
StorageLive(x);
s = S {f: T{}, g: T{}};
//                               { E s, e x }
//                               unpack(E s)
//                               { E s.f, E s.g, e x }
x = move s.g;
//                               { E s.f, e s.g, R x }

StorageLive(y);
y = s;
```

## Example: Packing and Unpacking

```
struct T {}
struct S { f: T, g: T }
```

```
StorageLive(s);
StorageLive(x);
s = S {f: T{}, g: T{}};
//                               { E s, e x }
//                               unpack(E s)
//                               { E s.f, E s.g, e x }
x = move s.g;
//                               { E s.f, e s.g, R x }
```

```
StorageLive(y);
y = s;
// Error
// can't obtain (E s) from
// { E s.f, e s.g, R x, e y }
```

```
error[E0382]: use of partially moved value: `s`
  --> src/main.rs:7:13
6 |         let x = s.g;
  |         --- value partially moved here
7 |         let y = s;
  |           ^ value used here after partial move
= note: partial move occurs because `s.g` has type `T`,
       which does not implement the `Copy` trait
```

## Example: Packing and Unpacking

```
struct T {}
struct S { f: T, g: T }

StorageLive(s);
StorageLive(x);
s = S {f: T{}, g: T{}};
//                               { E s, e x }
//                               unpack(E s)
//                               { E s.f, E s.g, e x }
x = move s.g;
//                               { E s.f, e s.g, R x }

s.g = T{};

y = s;
```

# Example: Packing and Unpacking

```

struct T {}
struct S { f: T, g: T }

```

```

StorageLive(s);
StorageLive(x);
s = S {f: T{}, g: T{}};
//           { E s, e x }
//           unpack(E s)
//           { E s.f, E s.g, e x }
x = move s.g;
//           { E s.f, e s.g, R x }

```

```

s.g = T{};
//           { E s.f, E s.g, R x, e y }
//           pack(E s)
//           { E s, R x, e y }
y = s;
//           { e s, R x, R y }

```

Rule:

```

{E s.f, E s.g}
  pack(E s)
{E s}

```



## Example: Packing and Unpacking

```
struct T {}
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//                               { E s.f, E s.g, e x }
x = move s.g;
//                               { E s.f, e s.g, R x }

s.g = T{};
//                               { E s.f, E s.g, R x, e y }
//                               pack(E s)
//                               { E s, R x, e y }
y = s;
//                               { e s, R x, R y }
```

## Example: Packing and Unpacking

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StorageLive(s);  
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//           unpack(E s)  
//           { E s.f, E s.g, e x }  
x = move s.g;  
//           { E s.f, e s.g, R x }
```

```
StorageDead(s);
```

## Example: Packing and Unpacking

```
struct T {}
struct S { f: T, g: T }

StorageLive(s);
StorageLive(x);
s = S {f: T{}, g: T{}};
//           { E s, e x }
//           unpack(E s)
//           { E s.f, E s.g, e x }
x = move s.g;
//           { E s.f, e s.g, R x }

//           drop(e s.f)
//           { e s.f, e s.g, R x }
//           pack(e s)
//           { e s, R x }
StorageDead(s);
//           { R x }
```

# Join Points

```
fn test(b: bool, t: T) {
    if b {
        StorageLive(u);
        u = move t;
//      drop(u)
        StorageDead(u);
//
//      { R b, r t }
    } else {
//      { R b, R t }

    }
}
```

# Join Points

```
fn test(b: bool, t: T) {
    if b {
        StorageLive(u);
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    }
}
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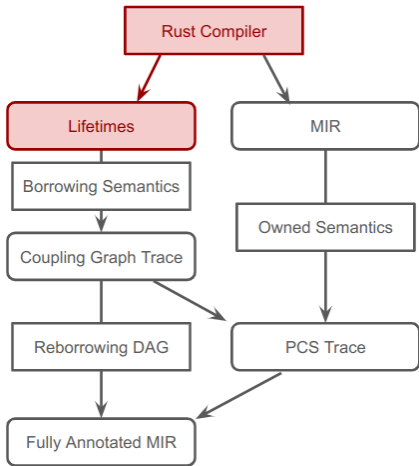
Which capabilities does the program have after the `if` statement?

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    } else {
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//      drop(t)
//      { R b, r t }
    }
//      { R b, r t }
}
```

Which capabilities does the program have after the `if` statement?

# Overview



# Mutable Borrows

```
let x = &mut z;
```

x will have some kind of control over z's memory for some time.



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# Mutable Borrows

```
let x = &mut z;
```

x will have some kind of control over z's memory for **some time**.

- E \*x, and no capability for z.
- Eventually, we relinquish E \*x and regain E z.

## 2<sup>nd</sup> notion of memory safety

```
let (x: &'a mut T) = (&mut z: &'bw0 mut T);
```

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let (x: &'a mut T) = (&mut z: &'bw0 mut T);
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- x needs to be usable for 'a.

## 2<sup>nd</sup> notion of memory safety

```
let (x: &'a mut T) = (&mut z: &'bw0 mut T);
```

- x needs to be usable for 'a.
- The borrow **can last for** 'bw0 before it must be given back.
  - Called an *invalidation* of 'bw0.

## 2<sup>nd</sup> notion of memory safety

```
let (x: &'a mut T) = (&mut z: &'bw0 mut T);
```

- x needs to be usable for 'a.
- The borrow can last for 'bw0 before it must be given back.
  - Called an *invalidation* of 'bw0.
- Require 'bw0 <: 'a (read 'bw0 *outlives* 'a)

## 2<sup>nd</sup> notion of memory safety

```
let (x: &'a mut T) = (&mut z: &'bw0 mut T);
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- x needs to be usable for 'a.
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### Memory Safety in Rust

Fixed point analysis over MIR.

Memory is **definitely not borrowed from** when invalidated.

⇒ No dangling pointers.

## 2<sup>nd</sup> notion of memory safety

```
let (x: &'a mut T) = (&mut z: &'bw0 mut T);
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- x needs to be usable for 'a.
- The borrow can last for 'bw0 before it must be given back.
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⇒ No **dangling pointers**.



```
struct T {}  
fn test(b: bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    if b {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x;  
    let usage_y = y;  
}
```

```
struct T {}  
fn test(b: bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    if b {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x; ←  
    let usage_y = y;  
}
```

x? Yes  
y? Yes  
t1? No: borrowed  
t2? No: borrowed

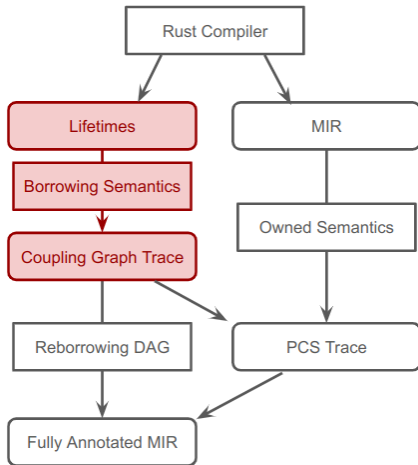
```
struct T {}  
fn test(b: bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    if b {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x;  
    let usage_y = y; ←  
}
```

x? No: moved out  
y? Yes  
t1? No: maybe borrowed  
t2? No: maybe borrowed

```
struct T {}  
fn test(b: bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    if b {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x;  
    let usage_y = y;  
}
```

x? No: moved out  
y? No: moved out  
t1? Yes  
t2? Yes ←

# Overview



## Setting: Mutable Borrows

### Prusti's Approach to Explaining Borrowed Data

- 1 Model borrow checker state as *DAG* of capabilities.
- 2 Describe *rules* in terms of that state.
- 3 Exploit *approximations* from the borrow checker.

# Capabilities for Borrows

Two complementary views:

- Temporary transfer of ownership
- Pointers with affine referent

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The second leads to a **repacking rule** for borrows:

- `let mut x = &mut t1;`  
`{E x} unpack(x) {e x, E (*x)}`



## Capabilities for Borrows

Two complementary views:

- Temporary transfer of ownership
- Pointers with affine referent

The second leads to a **repacking rule** for borrows:

- `let mut x = &mut t1;`  
`{E x} unpack(x) {e x, E (*x)}`
- `let tmp = x;`  
`{R tmp} unpack(tmp) {r tmp, E (*tmp)}`

# The Coupling Graph

- Directed Acyclic Hypergraph of Capabilities.

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- Outlives relationships become **abstract** capability exchanges.
  - Explains how to give back capabilities when borrows expire.

# The Coupling Graph

- Directed Acyclic Hypergraph of Capabilities.
- Outlives relationships become abstract capability exchanges.
  - Explains how to give back capabilities when borrows expire.
- Readily **approximated**, same way as the borrow checker.
  - Leaves correspond to **definitely not borrowed** places.
  - Rules for **coupling** edges (abstracting subgraphs at join points).

## Coupling Graph Rewrite Rules

```
// { E t1, e x, e z, e w }
```

```
x = &mut t1; // { E x, e z, e w }
```

{ E \*x } → { E t1 }

## Coupling Graph Rewrite Rules

```
// { E t1, e x, e z, e w }
```

```
x = &mut t1; // { E x, e z, e w }
```

$\{ E *x \} \rightarrow \{ E t1 \}$

```
z = move x; // { e x, E z, e w }
```

$\{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$

## Coupling Graph Rewrite Rules

```
// { E t1, e x, e z, e w }
```

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```
z = move x; // { e x, E z, e w }
```

$\{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$

```
unpack(z); // { e x, e z, E (*z), e w }
```

## Coupling Graph Rewrite Rules

```
// { E t1, e x, e z, e w }
```

```
x = &mut t1; // { E x, e z, e w }
```

$$\{ E *x \} \rightarrow \{ E t1 \}$$

```
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$$\{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$$

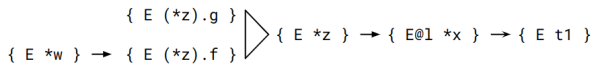
```
unpack(z); // { e x, e z, E (*z), e w }
```

```
unpack(*z); // { e x, e z, E (*z).f, E (*z).g, e w }
```

$$\begin{array}{l} \{ E (*z).g \} \\ \{ E (*z).f \} \end{array} \triangleright \{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$$



```
w = &mut (*z).f; // { e x, e z, E (*z).g, E w }
```



```
w = &mut (*z).f; // { e x, e z, E (*z).g, E w }
```

$$\{ E *w \} \rightarrow \{ E (*z).f \} \triangleright \{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$$

```
unpack(w); // { e x, e z, E (*z).g, e w, E (* w) }
```

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$$\begin{array}{l} \{ E (*z).g \} \\ \{ E *w \} \rightarrow \{ E (*z).f \} \end{array} \triangleright \{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$$

```
unpack(w); // { e x, e z, E (*z).g, e w, E (* w) }
```

```
/* w expires */ // { e x, e z, E (*z).g, E (*z).f, e w }
```

$$\begin{array}{l} \{ E (*z).g \} \\ \{ E (*z).f \} \end{array} \triangleright \{ E *z \} \rightarrow \{ E@1 *x \} \rightarrow \{ E t1 \}$$

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w = &mut (*z).f; // { e x, e z, E (*z).g, E w }
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```
/* z expires */ // { E t1, e x, e z, e w }
```

```
struct T {}  
fn test(b: bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    if b {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x;  
    let usage_y = y;  
}
```

## Coupling Graph “if” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \dots \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \dots \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$

## Coupling Graph “else” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$

## Coupling Graph “if” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \dots \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \dots \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$

## Coupling Graph “else” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$

## Coupled:

$$\begin{array}{l} \{E (*x: T)\} \longrightarrow \{'\theta\} \quad \begin{array}{c} \text{blue triangle} \\ \text{red triangle} \end{array} \quad \begin{array}{c} \text{white arrow} \\ \text{white arrow} \end{array} \quad \begin{array}{c} \text{dark blue triangle} \\ \text{dark blue triangle} \end{array} \quad \begin{array}{l} \{bw1\} \longrightarrow \{E (t2: T)\} \\ \{bw\theta\} \longrightarrow \{E (t1: T)\} \end{array} \end{array}$$

## Coupling Graph “if” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \dots \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \dots \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$

## Coupling Graph “else” branch:

$$\{E (*x: T)\} \longrightarrow \{'\theta\} \longrightarrow \{bw\theta\} \longrightarrow \{E (t1: T)\}$$
$$\{E (*y: T)\} \longrightarrow \{'1\} \longrightarrow \{bw1\} \longrightarrow \{E (t2: T)\}$$

## Coupled:

$$\begin{array}{l} \{E (*x: T)\} \longrightarrow \{'\theta\} \quad \begin{array}{c} \text{blue triangle} \\ \text{white arrow} \end{array} \quad \{bw1\} \longrightarrow \{E (t2: T)\} \\ \{E (*y: T)\} \longrightarrow \{'1\} \quad \begin{array}{c} \text{red triangle} \\ \text{white arrow} \end{array} \quad \{bw\theta\} \longrightarrow \{E (t1: T)\} \end{array}$$







Lifetime '0 expires:

- Consumes  $E (*x:T)$





Lifetime '0 expires:

- Consumes  $E (*x:T)$



Lifetime '1 expires

- Remainder of graph can expire
- Consumes  $E (*y:T)$ ;  $\{E (t1:T), E (t2:T)\}$  regained.

## Coupled Borrows as Loop Invariants

```
struct T {}  
fn test(f: fn() -> bool, mut t1: T, mut t2: T) {  
    let mut x = &mut t1;  
    let mut y = &mut t2;  
    while f() {  
        let tmp = x;  
        x = &mut (*y);  
        y = &mut (*tmp);  
    }  
    let usage_x = x;  
    let usage_y = y;  
}
```

# Coupled Borrows as Loop Invariants

```

struct T {}
fn test(f: fn() -> bool, mut t1: T, mut t2: T) {
    let mut x = &mut t1;
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    while f() {
        let tmp = x;
        x = &mut (*y);
        y = &mut (*tmp);
    }
    let usage_x = x;
    let usage_y = y;
}

```

$\{E (*x: T)\} \rightarrow \{ '0 \}$ 
 $\{E (*y: T)\} \rightarrow \{ '1 \}$ 
 $\{bw1\} \rightarrow \{E (t2: T)\}$ 
 $\{bw0\} \rightarrow \{E (t1: T)\}$

# Coupled Borrows as Loop Invariants

```

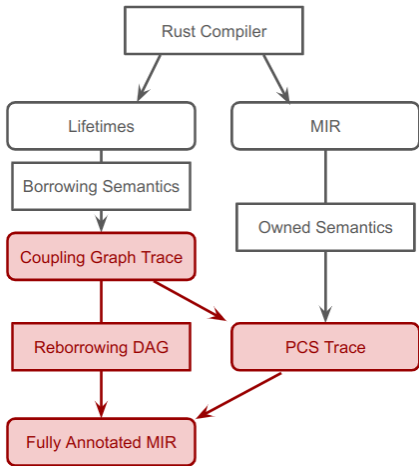
struct T {}
fn test(f: fn() -> bool, mut t1: T, mut t2: T) {
    let mut x = &mut t1;
    let mut y = &mut t2;
    while f() {
        let tmp = x;
        x = &mut (*y);
        y = &mut (*tmp);
    }
    let usage_x = x;
    let usage_y = y;
}

```



*stable under the loop body!*

# Overview



## The rest of the story

- Coupling graph edges govern subsets of a *reborrowing DAG*.



# The rest of the story

- Coupling graph edges govern subsets of a *reborrowing DAG*.
- Viper: Coupled edges can be packaged as magic wands
  - Apply wands at expiry or repackage of last coupled edge.
- Coupled edges at simple join points do not need magic wands.

## Shared Borrows, S subtyping

```
fn test(n: u32) {  
    let b0 = &n;  
    let b1 = &n;  
    /* clone n */  
    let _ = n;  
    let _ = b0;  
    let _ = b1;  
}
```

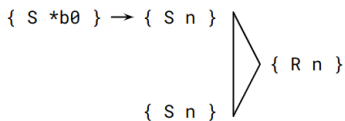
## Shared Borrows, S subtyping

```
fn test(n: u32) {  
    let b0 = &n;  
    let b1 = &n;  
    /* clone n */  
    let _ = n;  
    let _ = b0;  
    let _ = b1;  
}
```

```
{ R n, e b0, e b1 }
```

## Shared Borrows, S subtyping

```
fn test(n: u32) {
    let b0 = &n;
    let b1 = &n;
    /* clone n */
    let _ = n;
    let _ = b0;
    let _ = b1;
}
```

$$\{ S\ n, R\ b0, e\ b1 \}$$


- $\{ R\ b0 \}$  `unpack(b0)`  $\{ r\ b0, S\ (*b0) \}$
- $\{ R\ n, e\ b0 \}$  `b0 = &n;`  $\{ R\ b0, S\ n \}$

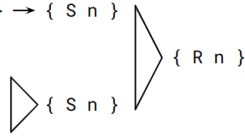
# Shared Borrows, S subtyping

```
fn test(n: u32) {
  let b0 = &n;
  let b1 = &n;
  /* clone n */
  let _ = n;
  let _ = b0;
  let _ = b1;
}
```

$$\{ S\ n, R\ b0, R\ b1 \}$$

$$\{ S\ *b0 \} \rightarrow \{ S\ n \}$$

$$\{ S\ *b1 \} \rightarrow \{ S\ n \}$$

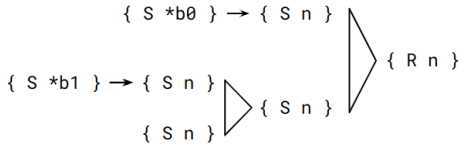
$$\{ S\ n \}$$


- $\{ R\ b0 \}$  `unpack(b0)`  $\{ r\ b0, S\ (*b0) \}$
- $\{ R\ n, e\ b0 \}$  `b0 = &n;`  $\{ R\ b0, S\ n \}$
- $\{ S\ n, e\ b1 \}$  `b1 = &n;`  $\{ S\ b0, S\ n \}$

# Shared Borrows, S subtyping

```
fn test(n: u32) {
  let b0 = &n;
  let b1 = &n;
  /* clone n */
  let _ = n;
  let _ = b0;
  let _ = b1;
}
```

{ S n, R b0, R b1 }



- { R b0 } unpack(b0) { r b0, S (\*b0) }
- { R n, e b0 } b0 = &n; { R b0, S n }
- { S n, e b1 } b1 = &n; { S b0, S n }
- { R n, - } - = clone n; { R n, - }
- { S n, - } - = clone n; { S n, - }

## Function Calls, Expiry Tools

```
fn test<'a, 'b, 'c, 'd>(x: &'a mut T, y: &'b mut T)
    -> (&'c mut T, &'d mut T)
where 'c <: 'a, 'c <: 'b, 'd <: 'b { /* ... */ }

let s = &mut t0;
let t = &mut t1;
let (m, n) = test(s, t);
```

---

<sup>1</sup>*Extended Support for Borrowing and Lifetimes in Prusti*

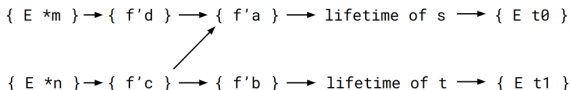
## Function Calls, Expiry Tools

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```

```
let s = &mut t0;
```

```
let t = &mut t1;
```

```
let (m, n) = test(s, t)
```



- Caller: Lorenz Gorse showed how to encode to Viper<sup>1</sup>.

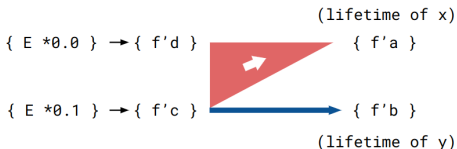
<sup>1</sup>*Extended Support for Borrowing and Lifetimes in Prusti*



## Function Calls, Expiry Tools

```
fn test<'a, 'b, 'c, 'd>(x: &'a mut T, y: &'b mut T)
    -> (&'c mut T, &'d mut T)
where 'c <: 'a, 'c <: 'b, 'd <: 'b { /* ... */ }
```

```
let s = &mut t0;
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let (m, n) = test(s, t)
```



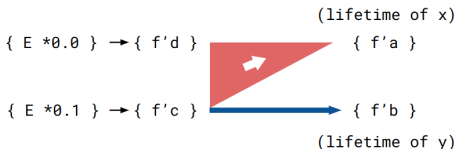
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- Callee: Polonius ensures we couple the above edges.

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## Function Calls, Expiry Tools

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fn test<'a, 'b, 'c, 'd>(x: &'a mut T, y: &'b mut T)
    -> (&'c mut T, &'d mut T)
where 'c <: 'a, 'c <: 'b, 'd <: 'b { /* ... */ }
```

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let s = &mut t0;
let t = &mut t1;
let (m, n) = test(s, t)
```



- Caller: Lorenz Gorse showed how to encode to Viper<sup>1</sup>.
- Callee: Polonius ensures we couple the above edges.
- *Expiry tools reify coupling graph expiry rules.*

<sup>1</sup>Extended Support for Borrowing and Lifetimes in Prusti

# Borrows in Structs

```
struct BorrowsLL<'a, S> {  
    data: &'a mut T,  
    next: Option<Box<BorrowsLL<'a, S>>>  
}  
let x: BorrowsLL<'a, T> = /* ... */;
```

## Borrows in Structs

```
struct BorrowsLL<'a, S> {  
    data: &'a mut T,  
    next: Option<Box<BorrowsLL<'a, S>>>  
}  
let x: BorrowsLL<'a, T> = /* ... */;
```

{ x'a } → (unbounded)

- Lazily add unpacking edges only on dereferences.

```
    { E (z: &'θ mut T) }  
{'θ} → { bwθ } → { E (t: T) }
```

```
    unpack(z);
```

```
    { e (z: &'θ mut T), E (*z: T) }  
{ E (*z: T) } → {'θ} → { bwθ } → { E (t: T) }
```

## Borrows in Structs

```
struct BorrowsLL<'a, S> {  
    data: &'a mut T,  
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## Borrows in Structs

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struct BorrowsLL<'a, S> {  
    data: &'a mut T,  
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}  
let x: BorrowsLL<'a, T> = /* ... */;
```

{ x'a } → (unbounded)

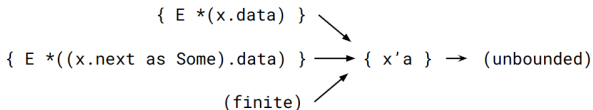
- Lazily add unpacking edges only on dereferences.
- { E x } unpack(x) { E x.data, E x.next }

# Borrows in Structs

```

struct BorrowsLL<'a, S> {
    data: &'a mut T,
    next: Option<Box<BorrowsLL<'a, S>>>
}
let x: BorrowsLL<'a, T> = /* ... */;

```



- Lazily add unpacking edges only on dereferences.
- $\{ E x \}$  `unpack(x)`  $\{ E x.data, E x.next \}$
- $\{ E x.data \}$  `unpack(x.data)`  $\{ e x.data, E *(x.data) \}$
- Coupling and PCS Joins ensure graph and PCS are always finite.



# Thank you for your attention!

Status/next steps:

- Implementation in Prusti
- Completeness proof
- Explore connections to other tools

Documentation and examples: Will be ready soon!